This exact expression for Q_L takes into account the frequency sensitivity of b. It should be noted that the center frequency is neither the geometric nor algebraic mean of the half-power frequencies [2].

Approximate equations for loaded Q are available that neglect the frequency sensitivity of b. In Mumford's classic paper on waveguide filters [3], the equation for loaded Q is

$$Q_L = \frac{1}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \cdot \left[-b\sqrt{b^2+4}\tan^{-1}\left(\frac{2}{b}\right)\right]. \quad (3)$$

Riblet [4] employs an expression for Q_L as

$$Q_L = \frac{(\pi - \phi)\cos\phi}{\sin^2\phi}. \quad (4)$$

Letting $\phi = \tan^{-1}(1/b)$, (4) becomes identical with (3).

Pritchard [5] and Fano and Lawson [6] compute the loaded Q from the following as

$$Q_L = \left(\frac{1+b^2}{4}\right)\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 \cdot \tan^{-1}\left(\frac{2b}{b^2-1}\right). \quad (5)$$

This equation—which has also appeared in Ragan (vol. 9 of the Radiation Laboratory Series) [7]—which presents the following simplified equation applicable to narrow-band filters when $b \gg 10$:

$$Q_L = \frac{\pi b^2}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2. \quad (6)$$

An approximate equation for loaded Q which does not employ transcedental functions will now be derived [8].

The lumped-circuit prototype of the doubly-loaded lossless waveguide cavity resonator is shown in Fig. 2. All circuit parameters have been normalized. Replacing the shunt loadings by equivalent series loadings, Fig. 3 is now applicable.

$$Q_L = \frac{\omega_0 l + 2x}{2r} = \frac{\omega_0 l'}{2r}. \quad (7)$$

For a short-circuited $\lambda_g/2$ transmission line, $\omega_0 l + 2x$ is, to a good approximation, equal to

$$\frac{\omega_0}{2Z_0}\frac{dX}{d\omega_0}\bigg|_{\omega_0} = \frac{\omega_0 L}{Z_0} = \omega_0 l' \quad (8)$$

where

ω_0 = angular resonant frequency
X = absolute reactance of waveguide transmission line
Z_0 = waveguide characteristic impedance.

Since $X = Z_0 \tan 2\pi l/\lambda_g$, then [9]

$$\frac{\omega_0}{2Z_0}\frac{dX}{d\omega}\bigg|_{\omega_0} = \frac{\pi}{2}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2 = \omega_0 l'. \quad (9)$$

To obtain r as shown in Fig. 3

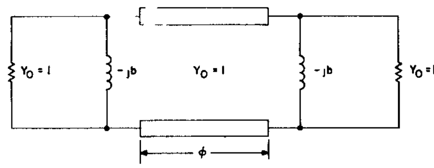

Fig. 1. Equivalent circuit of waveguide cavity resonator.

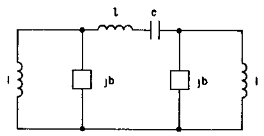

Fig. 2. Lumped-circuit prototype with shunt loadings.

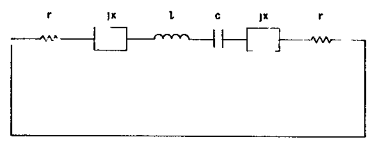

Fig. 3. Lumped-circuit prototype with series loadings.

let

$$z = r + jx = \frac{1}{1+jb} = \frac{1}{1+b^2} - \frac{jb}{1+b^2} \quad (10)$$

then

$$r = \frac{1}{1+b^2}. \quad (11)$$

Substituting (9) and (11) into (7), it can be shown that

$$Q_L = \frac{\pi}{4}\left(\frac{\lambda_{g0}}{\lambda_0}\right)^2(1+b^2). \quad (12)$$

Values of loaded Q for different normalized susceptances have been calculated using (2), (3), (5), (6), and (12). These are tabulated below, assuming $(\lambda_{g0}/\lambda_0)^2 = 2.08$:

			Q_L		
B	equation (2)	equation (3)	equation (5)	equation (6)	equation (12)
2	8.44	6.93	5.79	6.56	8.16
4	28.8	24.9	22.9	26.2	27.8
6	61.8	55.7	54.5	59.0	60.4
8	108	99.8	98.1	105	106.5
10	166	156.5	154	164	165
12	238	226	226	236	237

It can be seen that the approximate equation (12) provides the closest results when compared to exact equation (2). Errors of less than 3.3 percent will be incurred for values of $b \geqq 2.0$. When $b \geqq 10.0$, this error is reduced to a maximum of 0.6 percent. It should be noted that (2), (3), (5), (6), and (12) are applicable to the singly-loaded waveguide cavity by replacing the factor $\frac{1}{4}$ by $\frac{1}{2}$.

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Control of Resonant Frequency of YIG-Disk Filter by Doublet Tuning

I. Introduction

Magnetically tunable filters generally use ferrimagnetic resonators in the shape of a sphere. The most common ferrimagnetic material is single-crystal yttrium iron garnet (YIG), which has a saturation magnetization $4\pi M_s$ of approximately 1750 Oe. YIG spheres are satisfactory at frequencies down to almost 2 GHz, but doping is required (for instance, with gallium) to obtain high-Q resonance at lower frequencies. Since it is difficult to control the $4\pi M_s$ of Ga YIG, tuning adjustments have to be provided in multi-resonator filters, usually by a rotation of one of the Ga YIG spheres on a dielectric rod, changing the angles between its crystalline axes and the applied magnetic field.[1] The lower the operating frequency of the filter, the more doping is required, and the greater the variation in $4\pi M_s$ from resonator to resonator. By using disks rather than spheres, less doping is required.[2] However, it is more difficult to adjust the resonant frequency of a disk by rotation, than it is for a sphere. Another technique was therefore developed, using a pair of disks with adjustable spacing between them for each resonator; the spacing is adjusted to control the resonant frequency of the composite resonator.

In order to realize a multi-resonator bandpass filter with one Ga YIG disk per resonator, one must find a set of disks whose resonant frequencies at a given magnetic field, including the effects of shape and saturation, are sufficiently matched.[2] For example, in ordering disks (e.g., from Airtron, with nominal $4\pi M_s = 1000$ Oe), a tolerance less than ±10

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[1] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures.* New York: McGraw-Hill, 1964.
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Oe may not be specified, although in a shipment of say, five disks, one may with luck receive three with $4\pi M_sN_z$ values within only ± 5 Oe of the mean. What one would like to have is a means of obtaining an arbitrarily close approach to equality (and thus fewer rejects) and such a means was suggested. The idea was to combine two disks of disparate $4\pi M_s$ values into a parallel doublet—probably with adjustable spacing—so that there might be a composite, or effective, $4\pi M_sN_z$ value which could more readily be made to match that of another doublet on the other side of the filter.

A preliminary single-frequency (2.1 GHz) test with two pure YIG disks at various fixed spacings (disk axes along the [111] crystal axis) did indicate a single narrow-linewidth "composite" resonance that could be tuned downward in field as much as 80 Oe as the disks were brought together. Since this amount and direction of tuning were theoretically justifiable, further studies seemed worthwhile, especially since the effect of a wideband variation in frequency on the tracking of doublets needs to be known. It should be borne in mind, however, that a "matched pair" of pure YIG disks will almost always be closer in $4\pi M_sN_z$ at the outset than a matched pair of Ga YIG disks.

II. Tests with a Doublet of Ga YIG Disks

Tests were performed in a 575-MHz bandstop filter, with a pair of Ga YIG disks mounted in dielectric so they would remain parallel while the spacing was varied continuously (see Fig. 1). The individual disks exhibited strong, narrowline resonances when alone, but the disk axes here were [110] crystalline axes—a fact which probably accounts for the dismal results when the disks were combined into a doublet. As the disks were brought closer together, only very feeble resonances were observed, except at one particular relative orientation. (Since the mechanical motion depended on a short screw thread, one disk rotated through about one turn as it moved away from the stationary disk, and the critical spacing may well correspond to one fortuitously favorable alignment of crystalline axes. Disk rotation should, however, be immaterial when the disk axis is a [111] crystal axis, as is the case with our pure YIG disks.) Over the range of spacings where an effective $4\pi M_sN_z$ value could be reliably identified, it did not seem to be varying with spacing to any useful extent, and the tests were stopped.

To be fair, higher frequencies should have been tried in order to remove the disks further from the unsaturated condition. However, the question of crystal-axis alignment in doublets when the disk axis is not [111] oriented should be settled first. For example, a test in which such disks undergo relative rotation without change in spacing is in order. Another unknown is how much two $4\pi M_sN_z$ values can differ before a composite resonance is no longer obtainable. Theoretically, two composite resonances should be possible, corresponding to parallel and antiparallel RF magnetic field lines in the planes of the disks. Only the former should be evident here, because the doublet is entirely on one side of a stripline conductor. The other mode of resonance should be evident in the case of a center conductor passing between the disks.

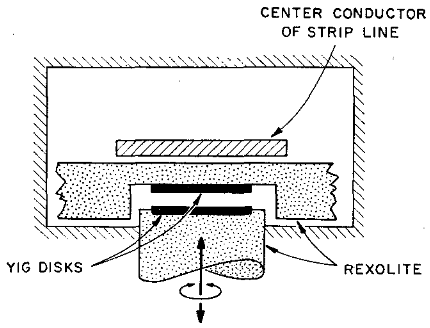

Fig. 1. Tuning mechanism of YIG-disk doublet in a bandstop filter mount.

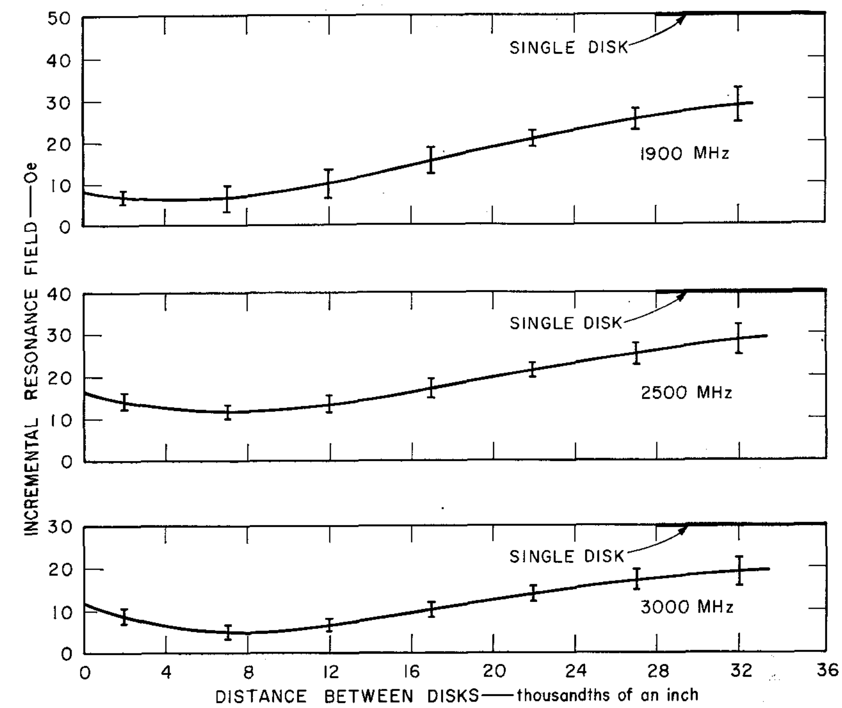

Fig. 2. Variation of resonant field for pure YIG doublet as a function of spacing, at frequencies of 1900, 2500, and 3000 MHz.

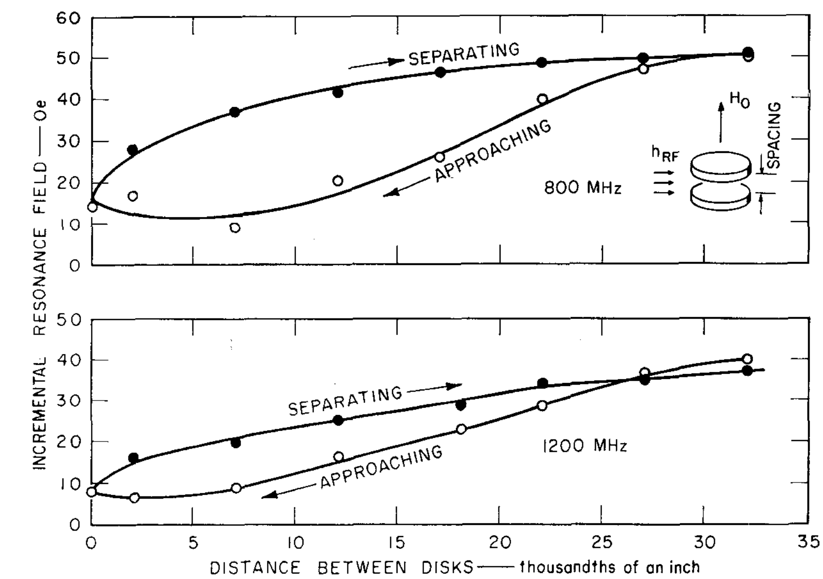

Fig. 3. Variation of resonant field for pure YIG doublet as a function of spacing, when the disks are approaching and when the disks are separating, at frequencies of 800 and 1200 MHz.

III. Tests with a Doublet of Pure YIG Disks

Another test with a pure YIG doublet was subsequently tried, using continuously variable spacing and a [111] orientation of the axes. This resulted in a smooth variation in resonant field with relative orientation of the two disks. The frequency range was extended to 3000 MHz, since higher frequencies are to undoped material what lower frequencies are to Ga YIG. A differential gaussmeter was used to show that the individual disks were initially very similar in $4\pi M_s N_z$ at a number of fixed frequencies. Since the procedure here involved inserting and removing the disks one at a time, in a bandstop-filter structure, orientations relative to the applied magnetizing field were probably not perfectly reproduced. Nevertheless, the measurements were repeatable within ± 3 Oe at 1200 MHz and above.

Resonances of this doublet were strong, and all data were clear cut and reproducible. In the octave 1500–3000 MHz, the variation of the effective $4\pi M_s N_z = H_{dc} - f_0/2.8$ with spacing was examined in detail and found to decrease by several tens of oersteds as the second disk was brought in from relatively far away to almost touching. This observation is consistent with theory. Most of the change occurs for spacings in the range 0.006 to 0.040 inch, for 0.150-inch diameter by 0.005-inch-thick disks. Figure 2 shows a few representative plots, at selected frequencies, of relative resonant field against disk spacing. The vertical limit marks represent a combination of experimental error and a "hysteresis" effect that will be explained soon by reference to Fig. 3.

Unfortunately, the observed change in effective $4\pi M_s N_z$ was neither constant nor even proportional to H_{dc} as the frequency varied. Actually, it dropped from about 45 Oe out of 2270 Oe at 1900 MHz, to about 25 Oe out of 2690 Oe at 3000 MHz. Thus, if we were to plot the resonant field against frequency for several disk spacings, all on one graph, then these several curves would not be parallel, though at the higher fields or frequencies they become more nearly parallel. This result means that if two unequal "inner" disks on opposite sides of a bandpass filter are "equalized" at one frequency by introducing two more "outer" disks at whatever spacing is appropriate to each side, they would not remain equalized at another frequency, and the filter would not be "tracking." As yet, no theory is available to apply to the aforementioned observed details; so far one can only predict approximately for the end-points, that is, for infinite and zero spacing.

An unusual phenomenon, observed at the lower frequencies, is exemplified in Fig. 3 for frequencies of 800 MHz and 1200 MHz. Below about 1500 MHz ($H_{dc} \approx 2100$ Oe) the resonant field for a given frequency and spacing is lower when the disks are approaching one another than when separating. This effect, which disappeared at higher fields and frequencies and therefore probably arises from some incomplete saturation condition, was always distinct and reproducible.

IV. Conclusions

Until doublets per se can be studied further, they cannot be used as adjustable resonators in bandpass filters. Instead, equality of $4\pi M_s N_z$ may have to depend on selecting pairs of "matched" disks from a large batch of disks.

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Bandstop Filter Formulas[1]

This correspondence summarizes a few simple formulas for single-resonator bandstop filters. We have found the formulas useful mainly in the tuning procedure or to determine the individual resonator Qs of multi-resonator bandstop filters [1]–[5]. These equations arose and were developed at the Stanford Research Institute during various applications; the exact formulas have not been published before.

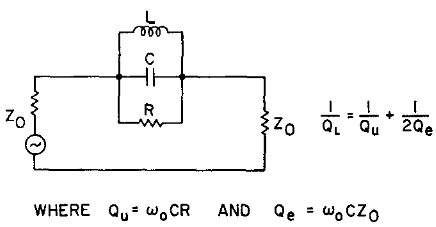

Fig. 1. Single-resonator bandstop filter.

Consider the single-resonator bandstop filter shown in Fig. 1, with the unloaded Q, Q_u, and external Q, Q_e, as defined in the figure. The source and load impedances (Z_0) are assumed to be equal. Let

$(L_A)_{\max}$ = maximum attenuation in dB of bandstop filter (with single resonator).

$V_{\max}$ = maximum input VSWR of bandstop filter (with single resonator).

$w_{x\text{-}dB}$ = fractional bandwidth between x-dB points on a guide wavelength basis. (The suffix 1 is used to emphasize that this is the bandwidth of a single cavity of possibly a multi-cavity filter in which the other cavities have been decoupled.)

$$A = \text{antilog}_{10}\left[\frac{(L_A)_{\max}}{20}\right] = 10^{(L_A)_{\max}/20}.$$

The resonator normalized slope parameter[1] is

$$\frac{b}{Y_0} = \frac{1}{2}\,\frac{Q_u}{A-1}.$$

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[1] These formulas resulted from research contracts for the U. S. Army Electronics Laboratories, Fort Monmouth, N. J.

Then,

$$A = \tfrac{1}{2}(V_{\max} + 1)$$

$$\frac{Q_u}{Q_e} = 2(A-1) = V_{\max} - 1$$

$$Q_u = \frac{1}{w_{3\text{-db}}}\sqrt{A^2 - 2}.$$

More generally,

$$Q_u = \frac{1}{w_{L_A\text{-}dB}}\sqrt{\frac{A^2 - 10^{L_A/10}}{10^{L_A/10} - 1}}$$

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Design Data for UHF Circulators

Introduction

Davies and Cohen [1] have described detailed calculations based upon the theoretical investigations of Bosma [2] into stripline circulator operation. The calculations yielded design curves for various modes of circulation but were applicable only for a particular relationship between center conductor width and ferrite radius. This correspondence describes an extension of their calculations to a wide range of stripline geometries, permitting greater freedom in circulator design.

Design Curves

The circulator geometry is depicted in Fig. 1, with the radius of the center conducting plane R being taken equal to the ferrite radius. Curves have been obtained as solutions of Bosma's circulation equations with the aid of a digital computer, using a technique which separates the required data from the singularities associated with solutions.

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